The Kardashian Kernel

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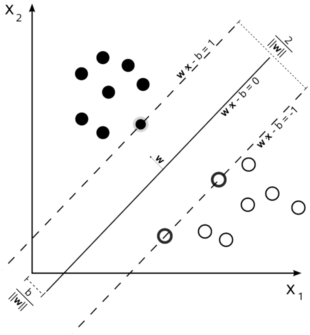
Outline

Motivation

- Kernel machines are popular
  - Have fancy math
  - They work well



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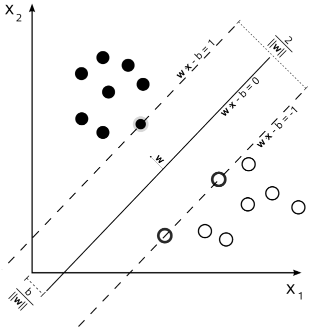
  - **(TODO)**



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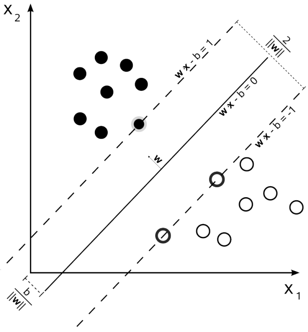
  - Have fancy math
  - They work well
- The Kardashians are popular
  - (**TODO**)
- Why not combine them?



Related work

- Kronecker product
- Krylov subspace methods
- Kolmogorov axioms
- Kalman Filters
- Kent distribution
- Karhunen-Loève Transform
- Keypoint retrieval w/ K-d tree search
- Kriging (AKA Gaussian process regression)
- Kohonen maps (AKA Self-Organizing Maps)
- K-grams
- K-folds
- K-armed bandits
- ...

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- Our approach: provably k-optimal, as our paper has significantly more k's and substantially more pictures of the Kardashians

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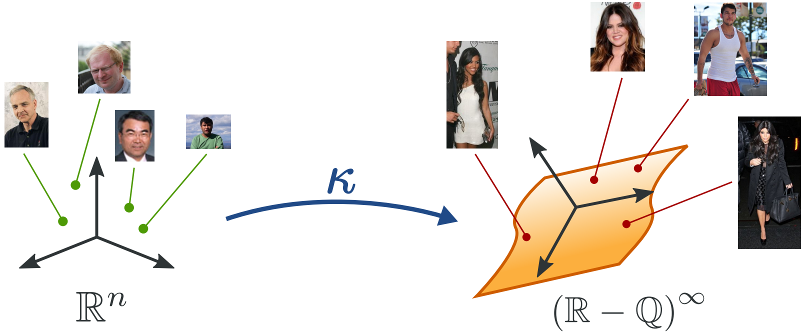
- Kernel trick (Mercer): $K_K(x, x') = \kappa(x)^T \kappa(x)$, with $\kappa : \mathcal{X} \to \mathfrak{K}$.
- can leverage the Kardashian Feature space without suffering the Kurse of Dimensionality.

The Kardashian Kernel Trick



$\kappa : \mathbb{R}^n \rightarrow Span\{$  ,  $\}$

$$K_K(x, x') = \kappa(x)^T \kappa(x')$$

On Some Issues Raised by the Kardashian Kernel

On Reproducing Kardashian Kernels

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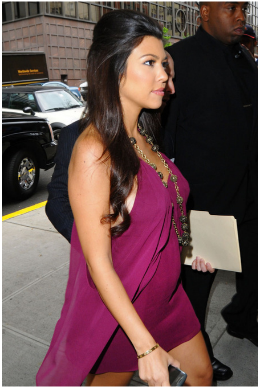
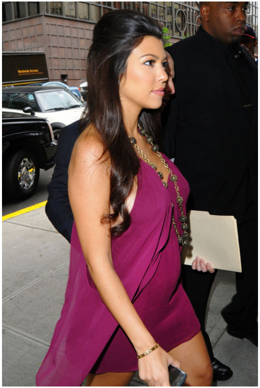


On Reproducing Kardashian Kernels

- Does K_K define a Reproducing Kernel Hilbert Space (RKHS)? i.e. are the Kardashians Reproducing Kernels?
- Only proven for case of Kourtney
- But prominent bloggers argue that it is also true for Kim



On Divergence Functionals

Crucial question: does the space induced by κ have structure that is advantageous to minimizing the f-divergences?

Theorem

$$\min_{w} = \frac{1}{n}\sum_{i=1}^{n}\langle w, \kappa(x_i)\rangle - \frac{1}{n}\sum_{j=1}^{n}\log\langle w, \kappa(y_j)\rangle + \frac{\lambda_n}{2}||w||_{\mathfrak{K}}^2$$

Proof.

Obvious by the use of the Jensen-Jenner Inequality. $\square$

Outline

Kardashian SVM problem setting

Regular Support Vector Machines (SVMs) are boring. We propose to solve the following optimization problem, which is subject to the Kardashian-Karush-Kuhn-Tucker (**KKKT**) Conditions:

$$\min_{\mathbf{w},\xi,\mathbf{b}} \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n}\xi_i$$

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such that

$$\begin{aligned} y_i(\mathbf{w}^T\kappa(\mathbf{x}_i) - \mathbf{b}) \geq 1 - \xi_i & \quad 1 \leq i \leq n \\ \xi_i \geq 0 & \quad 1 \leq i \leq n \\ \zeta_j = 0 & \quad 1 \leq j \leq m. \end{aligned}$$

Learning algorithm

- Standard approach: Kuadratic Programming (KP)

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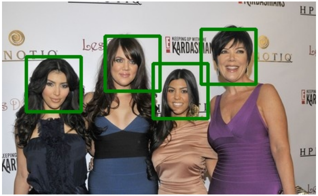
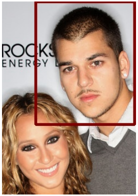
- But see Kurvature of optimization manifold
- Take advantage of geometry: Konvex-Koncave Procedure (KKP)

Experiment: Kardashian or Cardassian?



(a) Kardashian - (l. to r.) Kim, Khloé, Kourtney, Kris

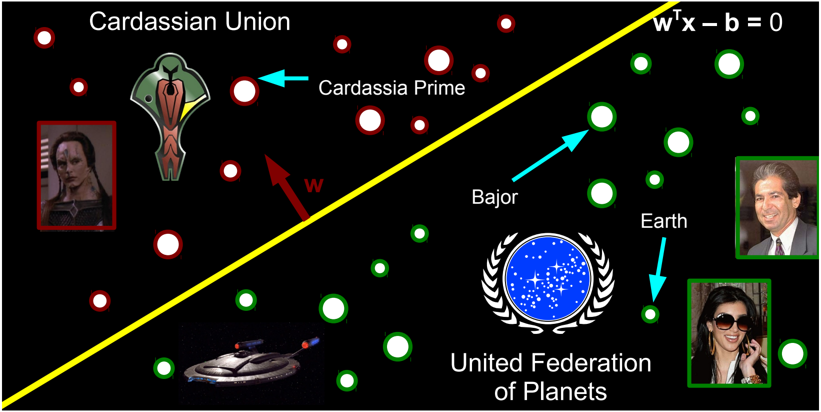
(b) Cardassian - (l. to r.) Gul Dukat, Elim Garak



(c) A failure case (or is it?): Kardashian - Rob

Our "Kardashian or Cardassian" dataset.

Schematic for Kardashian or Cardassian SVM



In the feature space $\mathfrak{K}$ induced by κ, the decision boundary between Cardassian and Kardashian lies approximately 5 light years from Cardassia Prime.

Graph Kardashiancian

Graph Kardashiancian

- The Graph Laplacian ℓ

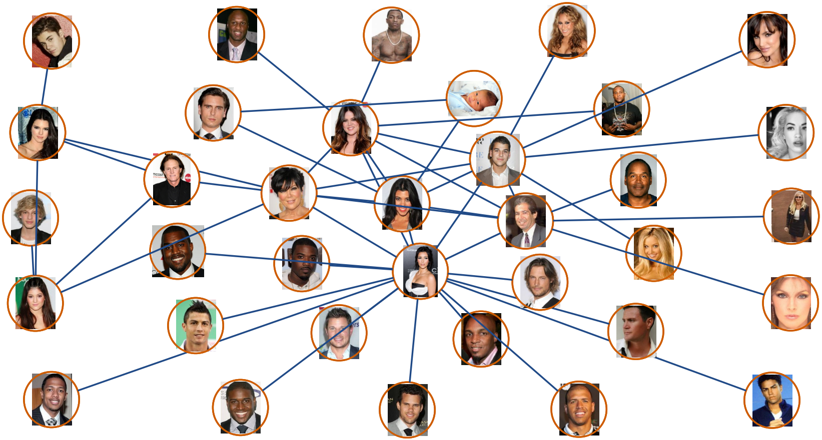
$$\ell_{i,j} := \begin{cases} \deg(v_i) & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise.} \end{cases}$$

Graph Kardashiancian

- The Graph Kardashiancian $\mathcal{K}$

$$\mathcal{K}_{i,j} := \begin{cases} \deg(v_i) & \text{if } i = j \\ -\kappa & \text{if } i \neq j \text{ and } v_i \text{ is Kardashian-adjacent to } v_j \\ 0 & \text{otherwise.} \end{cases}$$

Graph Kardashiancian



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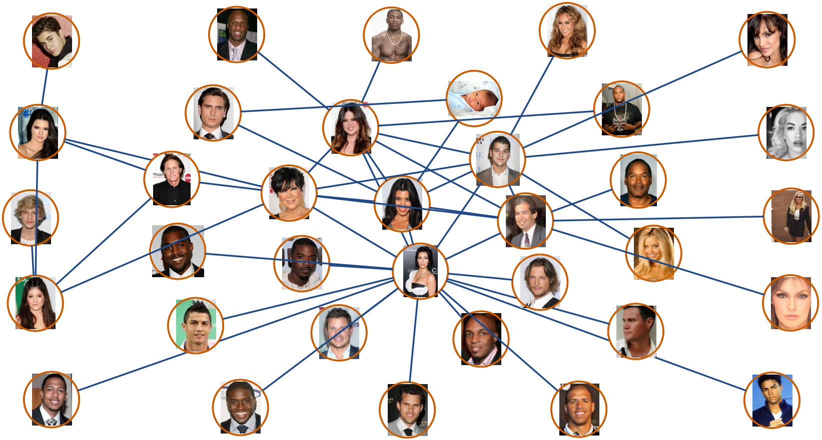

- Application: KardashianRank

Kardashian Kopula

Kardashian Kopula

- Powerful generalization of the Gaussian Copula

$$c_{\Sigma}(u) = \frac{1}{\sqrt{\det \Sigma}} \exp\left(-\frac{1}{2} \Phi^{-1}(u)^{T} \left(\Sigma^{-1} - \mathbf{I} \right) \Phi^{-1}(u) \right)$$

Kardashian Kopula

- Powerful generalization of the Gaussian Copula

$$c_{\Sigma}^{K}(u) = \frac{1}{\sqrt{\det \Sigma}} \exp\left(-\frac{1}{2}\mathbf{K}^{-1}(u)^{T}\left(\Sigma^{-1} - \mathbf{I}\right)\mathbf{K}^{-1}(u)\right)$$

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- Video illustrating the Kardashian Kopula (featuring rapper Ray J) may be found in the supplementary material

Outline

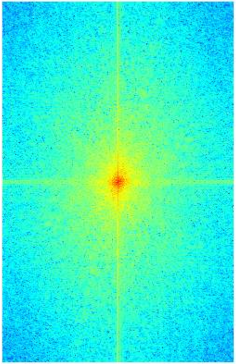
Celebrity-based Machine Learning

We have exhausted Kardashianity, but currently working on:

Celebrity-based Machine Learning

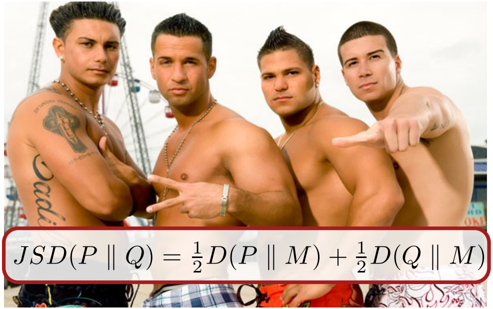
The Tila Tequila Transform (T_{T_T})

$$\xrightarrow{T_{T_T}(\mathcal{I})}$$



Celebrity-based Machine Learning

The Jensen-Shannon-Jersey-Shore (JS^2) divergence



$$JSD(P \parallel Q) = \frac{1}{2}D(P \parallel M) + \frac{1}{2}D(Q \parallel M)$$

A powerful generalization of The Kardashian-Kulback-Leibler (KKL) divergence

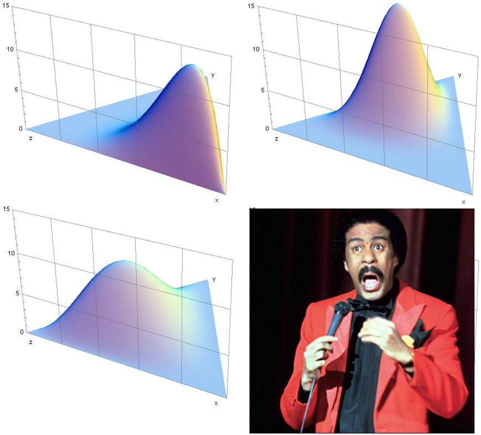
Celebrity-based Machine Learning

Jamie Lee Curtis Regularization

$$\min_{\beta(t)} \left(||y - \sum_{l=1}^{L} \mathbf{X}\beta(t)_l||_2^2 + \lambda||\beta(t) - \beta(t - 24h)||_2 \right)$$

Celebrity-based Machine Learning

The Richard Pryor Prior



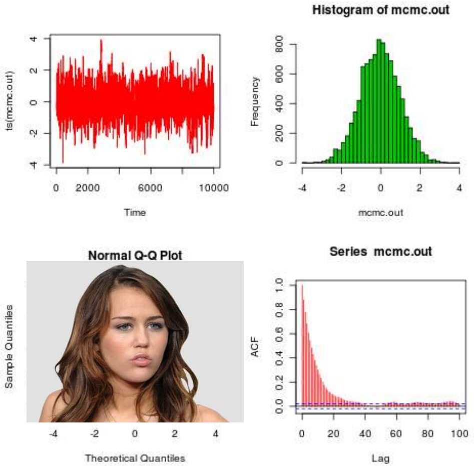
Celebrity-based Machine Learning

The Carrie Fisher Information Matrix

$\mathcal{I}(\theta) = \mathrm{E}\Big[\Big(\frac{\partial}{\partial\theta}\log$  $\Big)^2\Big|\,\theta\Big]$

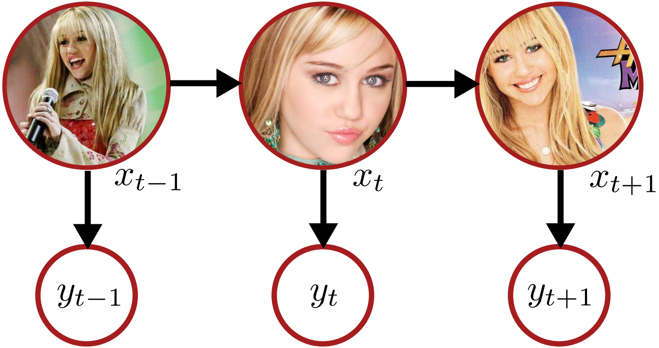
Celebrity-based Machine Learning

Miley Cyrus Markov Chain Monte Carlo (*MCMCMC*) methods for inference



Celebrity-based Machine Learning

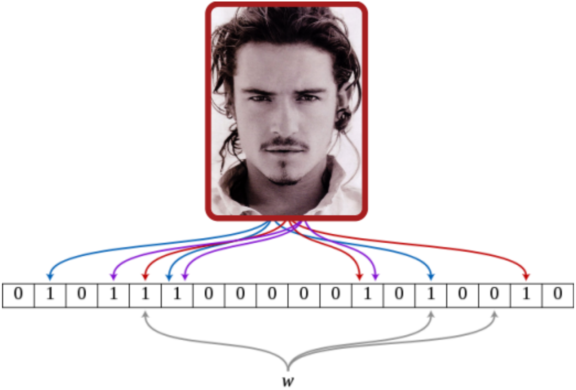
Hannah Montana Hidden Markov Models (*HMHMHMM*).



Train with MCMCMC for best of both worlds!

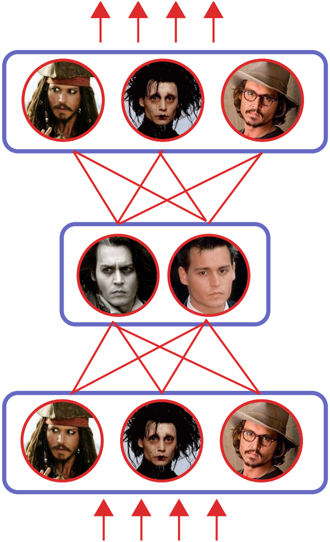
Celebrity-based Machine Learning

The Orlando Bloom Filter



Celebrity-based Machine Learning

Johnny Depp Belief Nets (*JDBN*s)



Thank you